Non Ideal MHD Effects in the PLUTO Code

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Introduction-Aims

Resistive effects play a key role in the disk-jet system of YSOs. Non ideal effects such as effective viscosity and magnetic reconnection are physical mechanisms governed by local parameters that we are still not aware of. Therefore, in order to have a better understanding of the jet-launching mechanisms, such phenomena have to be investigated numerically. The implementation and employment of resistive MHD codes is thus unavoidable.

We have included a new module in the PLUTO code [1] in order to take into account resistive effects.

Resistive MHD

Non ideal effects are taken into account by properly including resistive terms in the energy and the induction equation. This can be done without changing the conservative form of the equations:

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[(U + P_{tot})\mathbf{V} - \frac{1}{4\pi}(\mathbf{B}\cdot\mathbf{V})\mathbf{B} + \frac{1}{4\pi}\eta(\nabla\times\mathbf{B})\times\mathbf{B} \right] = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (-\mathbf{V}\times\mathbf{B} + \eta\nabla\times\mathbf{B}) = 0$$

At each time step, integration of the resistive terms can be done either explicitly or using the super time stepping technique.

Super Time Stepping Module

It is well known that parabolic problems constrain the time step to be proportional to the square of the cell size, whereas in hyperbolic problems (ideal MHD) the characteristic timesteps depends linearly on it. Therefore, in order to overcome the stability restriction (greater at higher resolution), we treat the diffusion terms using operator splitting techniques. In a first step we evolve the ideal MHD equations in a Godunov-type fashion; this update is then followed by a second step, where the energy and the magnetic field are evolved by taking into account the resistive contributions. During this step we employ STS [2], a technique that considerably accelerates explicit schemes.

A super step *t* is made, which is equal to the advection time step. This super step consists of *N* substeps $_j$, y requiring stability and optimality in the end of the super step, one gets the following relation for $_j$:

$$\tau_j = \Delta t_{expl}\left[(-1+\nu)\cos\left(\frac{2j-1}{N}\frac{\pi}{2} + 1 + \nu\right)\right]^{-1}$$

where t_{expl} is the standard explicit timestep and a stability parameter. When is sufficiently small it can be shown that:

$$\Delta T \underset{\nu\to 0}{\rightarrow} N^2 \Delta t_{expl}$$

Therefore, STS can be up to *N* times faster than the standard explicit scheme.

Current Status

Currently, integration of resistive terms works in one, two and three dimensions in all systems of coordinates. The implementation is compatible with both the 8-wave formulation (Powell) and Flux Constraint Transport (Balsara) for controling the divergence of the magnetic field.

The code is available from **http://plutocode.to.astro.it**

References

[1] Mignone et al. (Submitted)

[2] Alexiades et al., 1996, Com. Num. Meth. Eng., 12, 31

[3] O' Sullivan, Downes, 2006, MNRAS, 366, 13290

Code Validation

In the following, selected tests are being presented and compared with analytical solutions. We consider the diffusion of the magnetic field in a motionless plasma; in such cases the induction equation becomes:

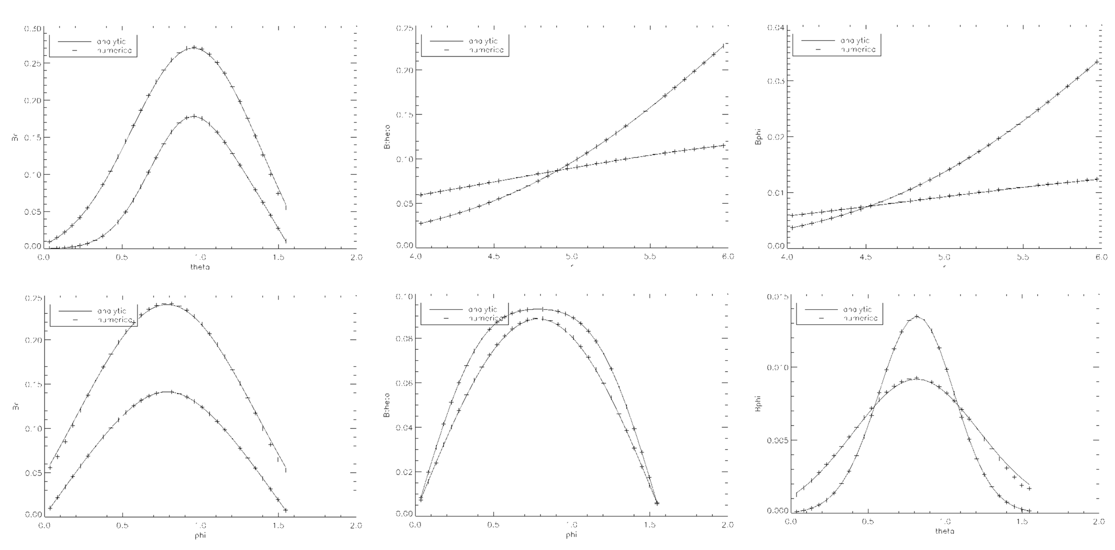
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta \nabla \times \mathbf{B})$$

Note that resistivity can have different values in each direction. The analytical solution in cartesian coordinates is

$$B_x = \frac{1}{t} exp(-\frac{y^2}{4\eta_z t}) exp(-\frac{z^2}{4\eta_y t})$$

The other components are found by cycling permutation of the coordinates.

• Explicit: 3comp., 3D, diff. test in spherical coordinates with constant resistivity

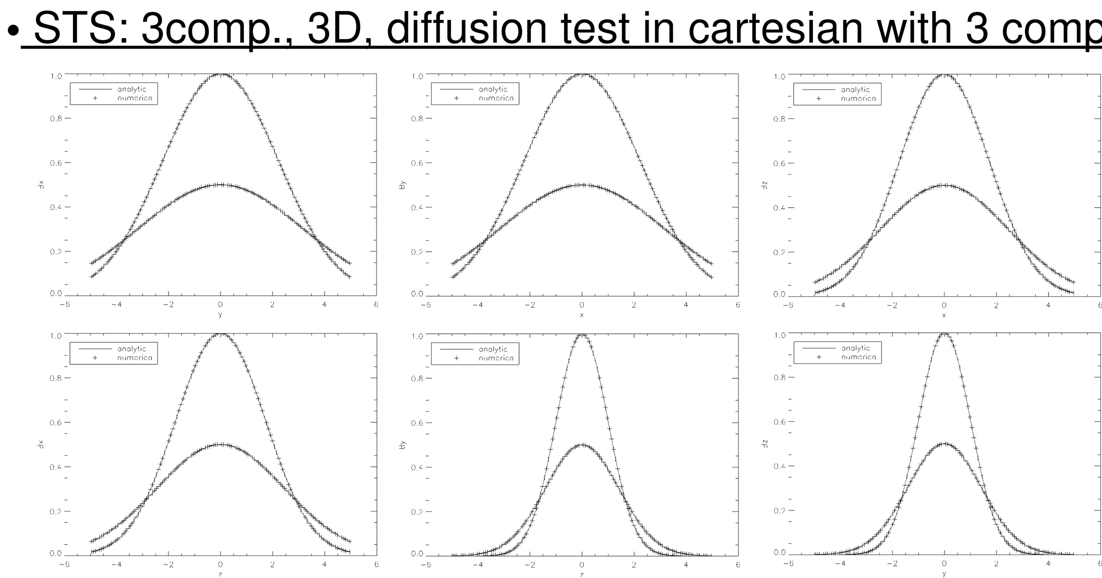


The 3 components of the magnetic field are plotted for the above solution rewritten in spherical coordinates. The simulation runs from t = 1.0 to t = 3.0. The resolution is 32^3

Resistivity is constant in all directions and equal to 1.0.

The maximum of the magnetic fields is centered in (5.0,5.0,5.0) in cartesian.

• STS: 3comp., 3D, diffusion test in cartesian with 3 components of resistivity



On the left, all the components of the magnetic field are plotted for the above solution of the induction equation in t = 1.0 and t = 2.0. The resolution is 128^3

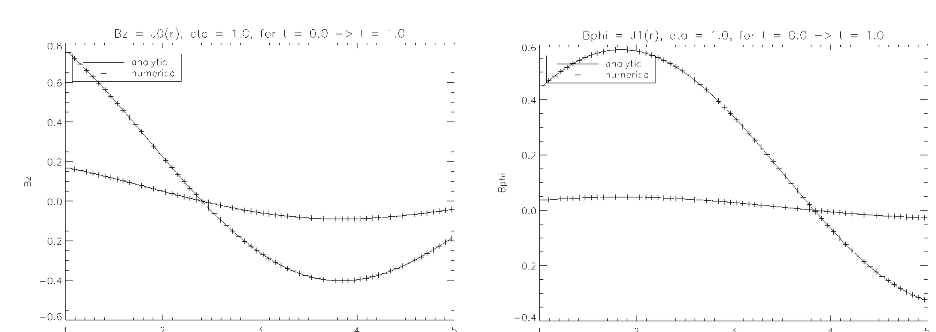
Resistivity is:

$_x$ = 0.5

$_y$ = 1.5

$_z$ = 2.5

• Comparison of Explicit and STS: 3component, 2.5D, diffusion test in cylindrical with constant resistivity.



The solution of the induction equation tested here are the Bessel functions $J_0(r)$ and $J_1(r)$ for B_z and B respectively. The resolution used is 512^2.

Identical results but, in this problem the STS module is more than 10 times faster.

Discussion and Future Plans

• The Super Time Stepping technique can retain stability and robustness while being many times faster than the standard explicit time integration. The only limitation is essentially imposed by the advective time scale of the problem.

• However, fine tuning of the nu-parameter may be necessary (depending on the problem) at higher resolutions.

• The presented technique is valid for parabolic terms in general and, as such, may be adopted for other physical processes, such as, viscosity, thermal conduction and radiation effects.

• By taking advantage of the computational framework provided by the PLUTO code, we are about to investigate the jet-launching problem in both the ideal and non-ideal regimes.

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