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The Paschen-Back effect on fine and hyperfine structure: impact on polarized spectra of Ap and Bp stars

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Abstract. We study how the Paschen-Back effect and hyperfine structure affect the Stokes parameters profiles of spectral lines formed under typical conditions of magnetic Ap and Bp stars. Assuming a dipolar magnetic geometry, we find remarkable effects for the sample lines considered.

1. Introduction

Atmospheres of magnetic stars of the Main Sequence are usually studied by analyzing the intensity and polarization profiles of one or a few spectral lines. Such data are then interpreted in terms of the Zeeman effect.

Nowadays, modern instruments allow us to obtain intensity and polarization profiles over wide spectral intervals, containing thousands of lines. When the profiles of the four Stokes parameters of all the lines contained in a spectral interval are to be interpreted, it is necessary to tackle two problems related to the process of line formation. On the one hand, in the presence of strong fields such as those of Ap and Bp stars, several spectral lines should better be described in terms of the Paschen-Back effect rather than the usual Zeeman effect. On the other hand, the presence of hyperfine structure can significantly alter the Stokes parameters profiles, especially for heavy elements.

The two problems are quite similar from the physical point of view. For each value of the magnetic field modulus, the fine-structure (or hyperfine-structure, resp.) Hamiltonian and the magnetic Hamiltonian (describing the interaction of the atom with the magnetic field) should be diagonalized simultaneously. The diagonalization must be performed for both the upper and lower term (or level, resp.) involved in the transition of interest. From the eigenvalues and eigenvectors it is then possible to obtain the splittings and strengths of the

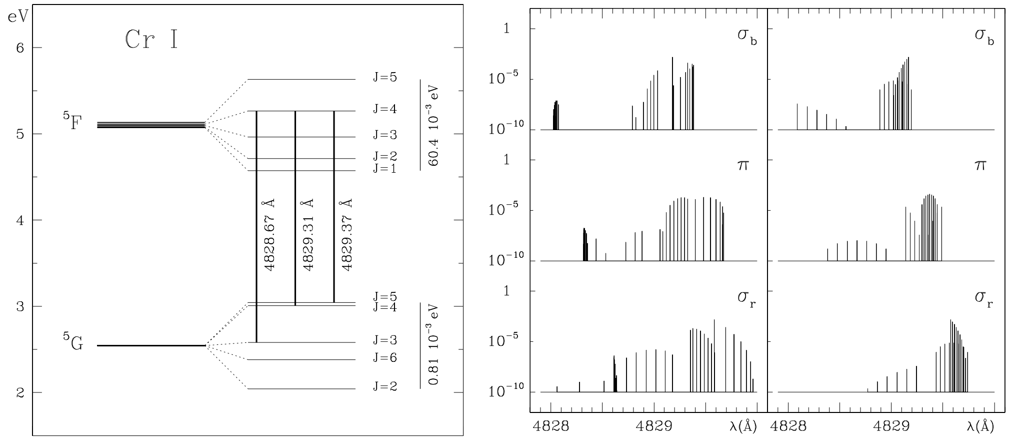

Figure 1. Left: the Cr I multiplet considered in this paper; the three lines of interest are labelled by their wavelength in air. Right: the corresponding pattern for $B = 20\,\mathrm{kG}$, taking into account the Paschen-Back effect (left), and under the Zeeman-effect approximation (right). Atomic data are from Moore (1971).

various components of the spectral line. It turns out that such components can still be divided, as in the usual Zeeman effect, into π and σ.

Once the pattern (splittings and strengths) has been calculated, the Stokes parameters profiles of the radiation coming from a given point of the stellar surface (characterized by a given magnetic field vector) can be obtained by solving the radiative transfer equation. Since the magnetic field changes on the stellar surface, the above calculations should be repeated for a sufficient number of points, and the Stokes parameters profiles should finally be integrated on the visible hemisphere.

2. Basic theory and application to two sample lines

Consider first the Paschen-Back effect in a fine-structured atom. Under the assumption of L-S coupling, the matrix elements of the magnetic Hamiltonian H_B for a given term, expressed on the basis of the energy eigenvectors, are given by

$$\langle \beta LSJM | H_B | \beta LSJ'M' \rangle = \delta_{MM'} \; \mu_0 B \Bigg[M \, \delta_{JJ'} \; + \; (-1)^{J+J'+L+S+M} \\ \times \sqrt{(2J+1)(2J'+1)S(S+1)(2S+1)} \begin{pmatrix} J & J' & 1 \\ -M & M & 0 \end{pmatrix} \begin{Bmatrix} J & J' & 1 \\ S & S & L \end{Bmatrix} \Bigg], \quad (1)$$

where J and M are the quantum numbers of the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ and of its projection on the quantization axis (coincident with the direction of the magnetic field $\vec{B}$), β denotes the electronic configuration, μ_0 is the Bohr magneton, and δ_{ab} is the Kronecker delta-function. The matrix elements of the fine-structure Hamiltonian $H_{\rm fs}$ are diagonal and can generally

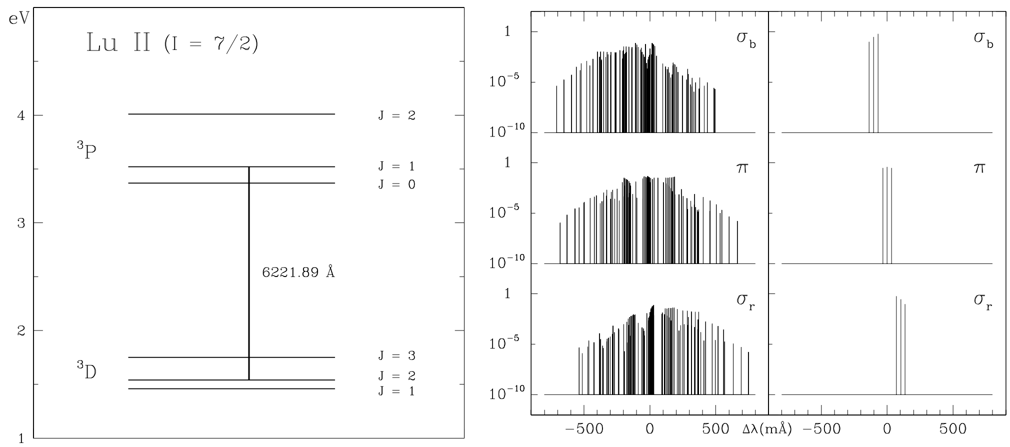

Figure 2. Left: energy levels for the Lu II line considered in this paper. Right: the corresponding pattern for $B = 5\,\mathrm{kG}$, taking into account (left) and neglecting (right) hyperfine structure. Values of the $\mathcal{A}$ and $\mathcal{B}$ constants are from Brix & Kopfermann (1952), values of Landé factors from Martin, Zalubas, & Hagan (1978).

be written in the form

$$\langle \beta LSJM | H_{\rm fs} | \beta LSJM \rangle \equiv E_{\beta LS}(J) = \frac{1}{2}\,\zeta\,\Big[J(J+1) - L(L+1) - S(S+1)\Big], \quad (2)$$

where the quantity ζ depends on the term considered. In any case, the energies of the different J-levels, $E_{\beta LS}(J)$, can be found from tables of atomic data.

Next we consider the effect of the magnetic field on an atom with hyperfine structure (non-zero nuclear spin I), restricting attention to the case where the magnetic splitting is small compared to the fine-structure separations ($\mu_0 B \ll \zeta$). The matrix elements of the magnetic Hamiltonian for a given (αJ)-level are given by

$$\langle \alpha JIFf | H_B | \alpha JIF'f' \rangle = \delta_{ff'}\ \mu_0 B\ g_{\alpha J}\ (-1)^{J+I-f}$$
$$\times \sqrt{(2F+1)(2F'+1)J(J+1)(2J+1)}\ \begin{pmatrix} F & F' & 1 \\ -f & f & 0 \end{pmatrix} \begin{Bmatrix} F & F' & 1 \\ J & J & I \end{Bmatrix}, \quad (3)$$

where F and f are the quantum numbers of the total angular momentum $\vec{F} = \vec{J} + \vec{I}$ and of its projection on the quantization axis (again coincident with the direction of $\vec{B}$), and $g_{\alpha J}$ is the Landé factor of the (αJ)-level. The matrix elements of the hyperfine-structure Hamiltonian $H_{\rm hfs}$ are diagonal and can be written in the form

$$\langle \alpha JIFf | H_{\rm hfs} | \alpha JIFf \rangle = \mathcal{A}\,\frac{K}{2} + \mathcal{B}\Big[K(K+1) - \frac{4}{3}\,J(J+1)I(I+1)\Big], \quad (4)$$

where $K = F(F+1) - J(J+1) - I(I+1)$ and $\mathcal{A}$, $\mathcal{B}$ (the so-called magnetic dipole and electric quadrupole constants) depend on the specific (αJ)-level and can be found in the specialized literature.

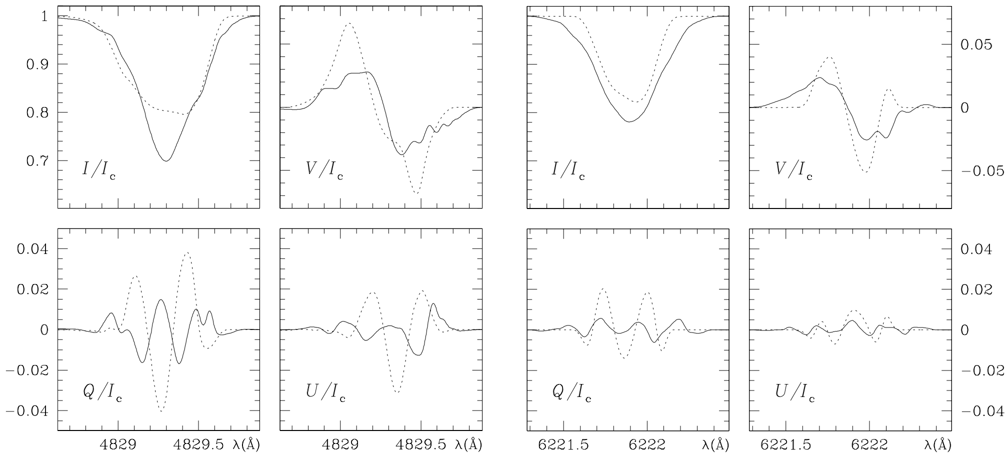

Figure 3. Stokes parameters profiles normalized to continuum intensity for Cr I $\lambda\,4829$ (left) and Lu II $\lambda\,6222$ (right) arising from a rotating, dipolar magnetic star. Full lines: taking into account the Paschen-Back effect (left), and hyperfine structure (right). Dashed lines: neglecting these effects.

As an example, we considered the lines $\lambda\,4829$ Å of Cr I and $\lambda\,6222$ Å of Lu II (see Figs. 1 and 2). As apparent from the panels on the right, the interaction between fine or hyperfine structure and the magnetic field produces large effects on the Zeeman pattern – note, however, the logarithmic scale.

For both lines we calculated, as illustrated above, the Stokes parameters profiles for a magnetic star, under the assumption of a Milne-Eddington model atmosphere (Planck function linear with optical depth, ratio of line to continuum absorption coefficient independent of optical depth), and a dipolar magnetic geometry. The curves in Fig. 3 refer to a dipole axis oriented at 45° from the line of sight and 30° from the positive Q direction, and to a polar field strength of 20 kG (left) and 5 kG (right). The projected equatorial velocity is $v_{\rm e}\sin i = 10\,{\rm km\,s^{-1}}$. The line Doppler width was set at 25.7 mÅ for $\lambda\,4829$ and 18.0 mÅ for $\lambda\,6222$, corresponding to a stellar temperature of about 8000 K.

In spite of the relatively large rotation velocity, there are indeed remarkable differences between full and dashed curves. The Paschen-Back effect and hyperfine structure are responsible both for an increase of the equivalent width and for a weakening of the polarization signals. Neglecting these phenomena may lead to a substantial overestimate of the chemical abundance, as well as to an incorrect determination of the field intensity and geometry.

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