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ON THE ATOMIC POLARIZATION OF THE GROUND LEVEL OF Na I

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ABSTRACT

In a recent Letter, we showed the remarkable result that the atomic alignment of the levels $P_{1/2}$ and $S_{1/2}$ of the D_1 line of Na I is practically destroyed in the presence of magnetic fields sensibly larger than 10 G, irrespective of the field direction. In this paper, we analytically demonstrate that this property is a consequence of the decoupling of the electronic and nuclear angular momenta $\boldsymbol{J}$ and $\boldsymbol{I}$ in the excited state $P_{3/2}$, which is achieved when the Zeeman splitting from the local magnetic field becomes much larger than the typical hyperfine separation for that level.

Subject headings: atomic processes — polarization — scattering — Sun: magnetic fields

1. INTRODUCTION

The observation and theoretical modeling of weak polarization signatures in spectral lines are opening a new window on the investigation of the weak magnetism of the solar atmosphere (see, e.g., the recent reviews by Stenflo 2001; Trujillo Bueno 2001; Trujillo Bueno & Manso Sainz 2002). To this aim, it is important to carefully investigate within the framework of the quantum theory of polarization (e.g., Landi Degl'Innocenti 1983) the observable effects of the atomic polarization of the energy levels involved in the line transitions of interest, including their subtle modification by the presence of magnetic fields.

In this respect, in a recent Letter (Trujillo Bueno et al. 2002a, hereafter Paper I), we reported on an interesting property of the polarizability of the levels of the D_1 line of Na I: in spite of the fact that those levels can both be aligned,[3] when proper account is taken of the additional quantum numbers introduced by the hyperfine structure (HFS) of Na I, the alignment is drastically reduced for fields larger than 10 G and practically vanishes for $B \gtrsim 100$ G, *irrespective* of the relative directions of the magnetic field and of the incident radiation. Accordingly, any contribution to the linear polarization in the core of D_1 that arises from atomic alignment is suppressed for magnetic fields sensibly larger than 10 G, so the only expected linear-polarization signal for such field strengths must be due to the transverse Zeeman effect (see Fig. 2 of Paper I; the reader should note how the Stokes Q signature of single-scattering events taking place in the presence of a vertical magnetic field changes from antisymmetric for $B < 10$ G to symmetric for $B \gtrsim 50$ G).

In Paper I, we were concerned mainly with a detailed calculation of the polarizability of the Na I levels and with the consequences it bears for our understanding of the magnetic field distribution and topology in the solar atmosphere. In the present work, we focus instead on the investigation of the atomic physics that is behind the polarization properties of those lines.

To this end, we follow the approach of Paper I and apply the quantum theory of line formation in the limit of complete frequency redistribution (CRD) and in the collisionless regime, as developed by Landi Degl'Innocenti (1983, 1984, 1985), to investigate the statistical equilibrium (SE) of an ensemble of Na I atoms illuminated by anisotropic radiation (see also Landolfi & Landi Degl'Innocenti 1985). The hypothesis of CRD corresponds to the requirement that the incident radiation field coming from the underlying photosphere and illuminating the scattering atoms be spectrally flat over an interval much larger than the energy separation between atomic levels whose wave functions sensibly overlap (leading to the phenomenon of quantum interferences). In the case of the D_1 and D_2 lines forming in the solar atmosphere, this is only a good assumption if we neglect the quantum interferences between the upper levels of D_1 and D_2. More specifically, these are interferences between the levels $P_{1/2}$ and $P_{3/2}$ pertaining to the same atomic term. Whereas the role of these so-called superinterferences is important for a correct interpretation of line polarization in the wings of D_1 and D_2, the line-core polarization of those lines, which was the subject of the investigation of Paper I, is expected to be largely unaffected by them.

In § 2 we summarize our qualitative description of the polarization properties of the levels of Na I (see Paper I) and introduce some useful new concepts and terminology. In § 3 we put those concepts on a more quantitative basis and provide an

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[3] Atomic *alignment* is a condition of population imbalances between the Zeeman substates of a level, such that the total population of substates with different values of $|M|$ are different. One speaks instead of atomic *orientation* when, for a given value of $|M|$, the substates labeled by M and $-M$ have different populations. See, e.g., Landi Degl'Innocenti (1984) or the recent review by Trujillo Bueno (2001).

algebraic proof that the alignment of the levels of D_1 is suppressed when a magnetic regime of complete decoupling of the angular momenta $\boldsymbol{J}$ and $\boldsymbol{I}$ is reached in the excited state $P_{3/2}$. Finally, in the conclusive section, we provide further arguments to illuminate this interesting phenomenon.

2. POLARIZABILITY OF THE Na I LEVELS: QUALITATIVE DESCRIPTION

The stable isotope of sodium has a nuclear spin $I = 3/2$; therefore, we must take into account the role of HFS in the solution of the SE problem of Na I. HFS was already indicated by Landi Degl'Innocenti (1998) as the only possible mechanism allowing for the existence of atomic alignment in the levels of the D_1 line. In fact, levels with total angular momentum $J = 1/2$ cannot be aligned, whereas both hyperfine levels $F = 1$ and 2, into which a level $J = 1/2$ splits in the coupling process with a nuclear spin $I = 3/2$, can be aligned.

For this reason, it is convenient to introduce the concept of *intrinsic polarizability* (IP), for those levels whose values of J allow the presence of atomic alignment, and of *extrinsic polarizability* (EP), for those levels that can only carry atomic alignment through the "internal" F quantum numbers, because of the presence of HFS. (What distinguishes the roles of J and F as quantum numbers, in this context, is the assumption we made at the beginning, that quantum interferences can only exist between different F levels, but not between different J levels.) In this sense, we can speak of EP only in the cases of $J = 0$ and $1/2$. Therefore, both levels of the D_1 line of Na I are EP, whereas the upper level of D_2 is IP, because $J = 3/2$.

This nomenclature has a direct link with the physics of the interaction processes of the atom with the incident radiation field. We speak of the IP of an atomic level when this level has the possibility of absorbing the multipole order $K = 2$ of the polarization tensor of the incident radiation field (Landi Degl'Innocenti 1983; see also Trujillo Bueno 2001), expressed in the irreducible spherical tensor representation, J^K_Q ($Q = -K, \ldots, K$). In particular, if we assume that the incident radiation field is unpolarized and has cylindrical symmetry around the local solar vertical through the scattering center, only the multipole orders $K = 0$ (intensity) and 2 (anisotropy) are present in the radiation-field tensor. In this case, it is found that an EP level can only absorb the multipole order $K = 0$, so there is no atomic polarization directly induced by the incident radiation field. Any atomic alignment ($K = 2$, in the irreducible spherical representation of the density matrix) that such a level can show—when proper account is taken of its substructure associated with HFS—can only come from the transfer of atomic alignment from other atomic levels that are instead IP. In the case of Na I, for instance, if the two levels of D_1 were isolated (i.e., not radiatively connected with other levels in the atom), no atomic alignment could be created, even accounting for the presence of HFS. Because of the presence of the upper level $P_{3/2}$ of D_2 in the SE problem of Na I, instead, transfer of atomic polarization from such an IP level to the lower level of D_1 can occur, via the radiative deexcitation associated with the formation of the D_2 line. Once EP has been created in the level $S_{1/2}$, this can be transferred via absorption processes to the upper level of D_1 as well. In our case, the two levels of D_1 manifest their EP because of the alignment induced onto the corresponding HFS levels, with $F = 1, 2$ (see Fig. 1 of Paper I; also Fig. 1 introduced below).

On the other hand, the transfer of atomic alignment from an IP level to an EP level can be inhibited under particular conditions. For the three-level model of the Na I atom considered here and for the prescribed radiation field, we determined that the atomic polarization in the two EP levels vanishes when the IP level $P_{3/2}$ reaches the regime of the complete Paschen-Back effect, in which the Zeeman splittings of the F levels due to the local magnetic field become much larger than the HFS separations between those levels. In this regime, the HFS coupling of the electronic and nuclear angular momenta $\boldsymbol{J}$ and $\boldsymbol{I}$ of the Na I atoms in the excited state $P_{3/2}$ is "relaxed" by the presence of the strong magnetic field, through the electronic Zeeman effect.[4] (To understand the meaning of such a decoupling process, we must observe that, in the regime of the complete Paschen-Back effect and assuming the direction of $\boldsymbol{B}$ as the quantization axis, J_z becomes a conserved quantity [rigorously, an element of the complete set of commuting observables of the atomic system], along with F_z. Because $I_z = F_z - J_z$ must be conserved as well, both M_J and M_I become good quantum numbers, so the eigenvectors of the atomic system take the form $|JM_J,\ IM_I\rangle$.)

The inhibition of the transfer of atomic alignment from an IP level to an EP level for increasing magnetic strengths is clearly illustrated by the results presented in Paper I. In Figure 1, we reproduce similar results. We calculated the atomic alignment of the levels of D_1 and D_2 for magnetic strengths between 10^{-4} and 10^3 G. A vertical field (i.e., aligned along the symmetry axis of the radiation cone from the photosphere illuminating the scattering atom) was chosen, in order to clarify that the obtained trend of the alignment against the magnetic field strength is *not* due to Hanle-effect depolarization. As we see, atomic alignment in the levels of D_1 is drastically reduced for fields larger than 10 G and practically vanishes for fields of the order of 100 G or larger. In Figure 2, we show analogous results for the atomic orientation ($K = 1$ in the irreducible spherical representation of the density matrix; see footnote 3 for a description of atomic orientation), for magnetic strengths between 10^{-3} and 10^4 G. Also in this case, the orientation of the levels of the D_1 line practically vanishes for $B \gtrsim 100$ G. (We note that for $B > 100$ G, the level $P_{3/2}$ approaches the regime of the complete Paschen-Back effect. In fact, for $B \sim 500$ G, the typical Zeeman splitting is already 10 times larger than the typical HFS separation for that level.)

On the other hand, as suggested by the work of Lehmann (1969) concerning optical-pumping processes in cadmium, a sufficient condition for the vanishing of atomic alignment in the EP level is that the HFS frequency separation of the IP level be negligible with respect to the Einstein A coefficient of the atomic transition. This condition is very general, as it holds regardless of the magnetic field strength (in particular, it is valid also for zero magnetic fields). In the case of Na I, the HFS frequency separation of the IP level $P_{3/2}$ is comparable to the Einstein A coefficient of the D_2 line.[5] For this reason, transfer of atomic

[4] The nuclear Zeeman effect is completely negligible in our picture, up to field strengths of the order of 10^5 G.

[5] Coincidentally, this implies that the regime of the complete Paschen-Back effect in the upper level also corresponds to the saturation regime of the Hanle effect for that level.

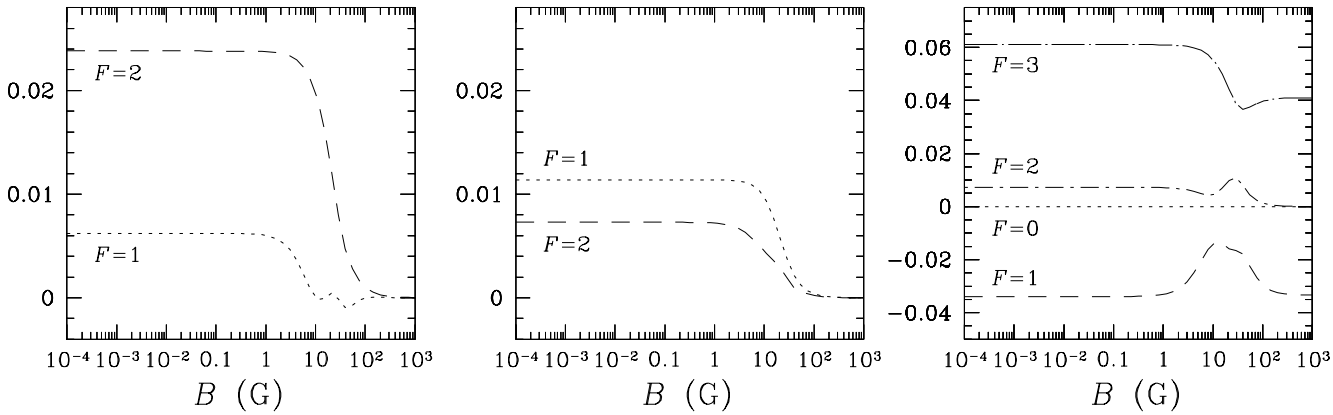

FIG. 1.—Fractional atomic alignment $\sigma_0^2(F) = \rho_0^2(F,\ F)/\rho_0^0(F,\ F)$ of the three lowest levels of Na I against the magnetic field strength. A vertical field and a height of 10″ of the scattering atoms above the solar surface are assumed. The kinetic temperature of the emitting plasma is $T = 6000$ K. These results show that the atomic alignment of the levels of D_1 is practically zero when $B \gtrsim 100$ G, even for vertical fields.

alignment from the IP upper level to the EP lower level is possible when $B < 100$ G, since $\boldsymbol{J}$ and $\boldsymbol{I}$ are still significantly coupled in the IP level $P_{3/2}$ (see Paper I; end of § 3).

These results suggest that the inhibition of the transfer of atomic alignment from an IP level to an EP level should be regarded as an aspect of the so-called principle of spectroscopic stability applied to the IP level: whenever the hyperfine structure of the IP level becomes negligible, whether because a magnetic field is present that is strong enough to reach the complete Paschen-Back regime for that level or because the HFS separation of that level is much smaller than its radiative width, the transfer of alignment from the IP level to the EP level is inhibited, so the EP level behaves as if the atomic HFS were not present at all. The reason for this is hidden in the complexity of the SE problem, and it is addressed in the following section.

3. POLARIZABILITY OF THE Na I LEVELS: ANALYTICAL DESCRIPTION

We consider an IP level with total electronic angular momentum J. We assume that this level can only interact with EP levels in the atom. Beyond this restriction, the atomic system can be arbitrary, so the following formalism also applies for atoms other than Na I. If a nuclear spin of angular momentum I is present, the density matrix for the IP level in the irreducible spherical tensor representation is (e.g., Landi Degl'Innocenti 1984)

$$ {}^{JI}\rho_Q^K(F,\ F') = \sum_{M_F M'_F} (-1)^{F-M_F} \sqrt{2K+1} \begin{pmatrix} F & F' & K \\ M_F & -M'_F & -Q \end{pmatrix} {}^{JI}\rho(FM_F,\ F'M'_F)\ . \tag{1}$$

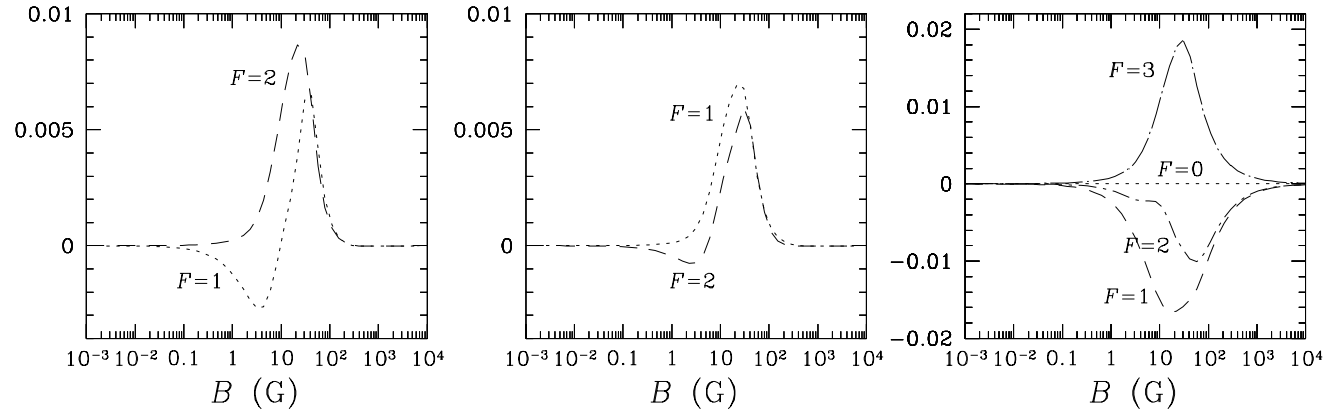

FIG. 2.—Fractional atomic orientation $\sigma_0^1(F) = \rho_0^1(F,\ F)/\rho_0^0(F,\ F)$ of the three lowest levels of Na I against the magnetic field strength (notice the different range of magnetic strengths with respect to Fig. 1). The scattering geometry and the plasma conditions are the same as in the calculation of Fig. 1. These results show that atomic orientation in the levels of D_1 is negligible for fields $B \gtrsim 100$ G.

We write explicitly

$$ {}^{JI}\rho(FM_F,\ F'M'_F) = \langle (JI)FM_F|\rho|(JI)F'M'_F\rangle \ , \tag{2} $$

where

$$ |(JI)FM_F\rangle = \sum_{M_J M_I} C(JM_J IM_I;\ FM_F)|JM_J,\ IM_I\rangle \ . \tag{3} $$

In the previous equation, $C(JM_J IM_I;\ FM_F)$ are Clebsh-Gordan coefficients, which can be expressed in terms of 3*j* symbols as

$$ C(JM_J IM_I;\ FM_F) = (-1)^{J-I+M_F}\sqrt{2F+1}\begin{pmatrix} J & I & F \\ M_J & M_I & -M_F \end{pmatrix} . \tag{4} $$

Substitution of equation (3) into equation (2) using equation (4) gives

$$ \begin{aligned} {}^{JI}\rho(FM_F,\ F'M'_F) = {} & (-1)^{M_F-M'_F}\sqrt{(2F+1)(2F'+1)} \\ & \times \sum_{M_J M_I M'_J M'_I} \begin{pmatrix} J & I & F \\ M_J & M_I & -M_F \end{pmatrix}\begin{pmatrix} J & I & F' \\ M'_J & M'_I & -M'_F \end{pmatrix}\langle JM_J,\ IM_I|\rho|JM'_J,\ IM'_I\rangle \ . \end{aligned} \tag{5} $$

We now make the assumption that the electronic spin and the nuclear spin are decoupled (or very weakly coupled) when the atom is in the IP level. As anticipated in the previous section, this can be the case if the HFS separation is much smaller than the natural width of that level or, in the presence of a magnetic field, if the level is in the regime of the complete Paschen-Back effect. In either case, the atomic density matrix for the IP level can be factored as

$$ \langle JM_J,\ IM_I|\rho|JM'_J,\ IM'_I\rangle = \rho(JM_J,\ JM'_J)\rho(IM_I,\ IM'_I) \ . \tag{6} $$

We introduce at this point the formalism of the irreducible spherical tensors for both $\rho(JM_J,\ JM'_J)$ and $\rho(IM_I,\ IM'_I)$,

$$ \rho(JM_J,\ JM'_J) = \sum_{K_J Q_J}(-1)^{J-M_J}\sqrt{2K_J+1}\begin{pmatrix} J & J & K_J \\ M_J & -M'_J & -Q_J \end{pmatrix}\rho^{K_J}_{Q_J}(J) \ , \tag{7a} $$

$$ \rho(IM_I,\ IM'_I) = \sum_{K_I Q_I}(-1)^{I-M_I}\sqrt{2K_I+1}\begin{pmatrix} I & I & K_I \\ M_I & -M'_I & -Q_I \end{pmatrix}\rho^{K_I}_{Q_I}(I) \ . \tag{7b} $$

Substitution of equation (6) into equation (5) using equations (7a) and (7b) gives

$$ \begin{aligned} {}^{JI}\rho(FM_F,\ F'M'_F) = {} & (-1)^{J+I-M'_F}\sqrt{(2F+1)(2F'+1)}\sum_{K_J Q_J}\sum_{K_I Q_I}\sqrt{(2K_J+1)(2K_I+1)}\rho^{K_J}_{Q_J}(J)\rho^{K_I}_{Q_I}(I) \\ & \times \sum_{M_J M'_J M_I M'_I}\begin{pmatrix} J & I & F \\ M_J & M_I & -M_F \end{pmatrix}\begin{pmatrix} J & I & F' \\ M'_J & M'_I & -M'_F \end{pmatrix}\begin{pmatrix} J & J & K_J \\ M_J & -M'_J & -Q_J \end{pmatrix}\begin{pmatrix} I & I & K_I \\ M_I & -M'_I & -Q_I \end{pmatrix} . \end{aligned} $$

Finally, this equation must be substituted into equation (1). We then obtain an expression that involves the contraction over all magnetic quantum numbers of a product of five 3*j* symbols. This contraction can be evaluated using, e.g., equation (14) of Varshalovich, Moskalev, & Khersonskii (1988, p. 456), yielding the expression

$$ \begin{aligned} {}^{JI}\rho^K_Q(F,\ F') = {} & (-1)^{K-Q}\sqrt{(2K+1)(2F+1)(2F'+1)}\sum_{K_J K_I}\sqrt{(2K_J+1)(2K_I+1)}\begin{Bmatrix} J & I & F \\ J & I & F' \\ K_J & K_I & K \end{Bmatrix} \\ & \times \sum_{Q_J Q_I}\begin{pmatrix} K & K_J & K_I \\ Q & -Q_J & -Q_I \end{pmatrix}\rho^{K_J}_{Q_J}(J)\rho^{K_I}_{Q_I}(I) \ . \end{aligned} \tag{8} $$

As a particular case, if nuclear polarization is absent ($K_I = Q_I = 0$), equation (8) reduces to

$$ {}^{JI}\rho^K_Q(F,\ F') = (-1)^{J+I+F'+K}\rho^0_0(I)\sqrt{\frac{(2F+1)(2F'+1)}{2I+1}}\begin{Bmatrix} J & J & K \\ F & F' & I \end{Bmatrix}\rho^K_Q(J) \ . \tag{9} $$

In this case, the (electronic) atomic polarization of the *J* level translates *directly* (i.e., with the same *K* and *Q*) into the atomic polarization of the $(F,\ F')$ pair.

3.1. *The Effect of Very Weak J-I Coupling on the SE Problem*

As an application of the former development, we consider a two-level atom $(J_u,\ J_l)$ endowed with HFS. Neglecting stimulated emission for simplicity, the SE equations for the two levels read (Landi Degl'Innocenti 1983, 1984, 1985)

$$\begin{aligned}\frac{d}{dt}{}^{J_uI}\rho^{K_u}_{Q_u}(F_u,\ F'_u) = &-i\sum_{F''_uF'''_u}\sum_{K'_uQ'_u}{}^{J_uI}\rho^{K'_u}_{Q'_u}(F''_u,\ F'''_u)N(F_uF'_uK_uQ_u;\ F''_uF'''_uK'_uQ'_u)\\ &-\sum_{F''_uF'''_u}\sum_{K'_uQ'_u}{}^{J_uI}\rho^{K'_u}_{Q'_u}(F''_u,\ F'''_u)R_E(F_uF'_uK_uQ_u;\ F''_uF'''_uK'_uQ'_u)\\ &+\sum_{F_lF'_l}\sum_{K_lQ_l}{}^{J_lI}\rho^{K_l}_{Q_l}(F_l,\ F'_l)T_A(F_uF'_uK_uQ_u;\ F_lF'_lK_lQ_l)\ ,\end{aligned}\tag{10}$$

$$\begin{aligned}\frac{d}{dt}{}^{J_lI}\rho^{K_l}_{Q_l}(F_l,\ F'_l) = &-i\sum_{F''_lF'''_l}\sum_{K'_lQ'_l}{}^{J_lI}\rho^{K'_l}_{Q'_l}(F''_l,\ F'''_l)N(F_lF'_lK_lQ_l;\ F''_lF'''_lK'_lQ'_l)\\ &-\sum_{F''_lF'''_l}\sum_{K'_lQ'_l}{}^{J_lI}\rho^{K'_l}_{Q'_l}(F''_l,\ F'''_l)R_A(F_lF'_lK_lQ_l;\ F''_lF'''_lK'_lQ'_l)\\ &+\sum_{F_uF'_u}\sum_{K_uQ_u}{}^{J_uI}\rho^{K_u}_{Q_u}(F_u,\ F'_u)T_E(F_lF'_lK_lQ_l;\ F_uF'_uK_uQ_u)\ .\end{aligned}\tag{11}$$

To understand how atomic polarization is created in an EP level, assuming that the other level is IP, we must explicitly consider the expressions of the transfer rates for absorption and spontaneous emission processes, respectively,

$$\begin{aligned}T_A(F_uF'_uK_uQ_u;\ F_lF'_lK_lQ_l) = &(2J_l+1)B_{J_lJ_u}\sqrt{(2F_u+1)(2F'_u+1)}\\ &\times\sum_{K_rQ_r}\sqrt{3(2K_u+1)(2K_l+1)(2K_r+1)}\begin{pmatrix}K_u & K_l & K_r\\ -Q_u & Q_l & -Q_r\end{pmatrix}J^{K_r}_{Q_r}(\omega_{ul})\\ &\times(-1)^{F'_l-F_l+K_l+Q_l}\sqrt{(2F_l+1)(2F'_l+1)}\begin{Bmatrix}F_u & F_l & 1\\ F'_u & F'_l & 1\\ K_u & K_l & K_r\end{Bmatrix}\begin{Bmatrix}J_u & J_l & 1\\ F_l & F_u & I\end{Bmatrix}\begin{Bmatrix}J_u & J_l & 1\\ F'_l & F'_u & I\end{Bmatrix},\end{aligned}\tag{12}$$

$$\begin{aligned}T_E(F_lF'_lK_lQ_l;\ F_uF'_uK_uQ_u) = &\delta_{K_lK_u}\delta_{Q_lQ_u}(2J_u+1)A_{J_uJ_l}\sqrt{(2F_l+1)(2F'_l+1)}\\ &\times(-1)^{F'_l+F'_u+K_l+1}\sqrt{(2F_u+1)(2F'_u+1)}\begin{Bmatrix}F_l & F'_l & K_l\\ F'_u & F_u & 1\end{Bmatrix}\begin{Bmatrix}J_u & J_l & 1\\ F_l & F_u & I\end{Bmatrix}\begin{Bmatrix}J_u & J_l & 1\\ F'_l & F'_u & I\end{Bmatrix}.\end{aligned}\tag{13}$$

The relaxation rate due to spontaneous emission R_E is completely diagonal, so it can only relate each of the elements ${}^{JI}\rho^K_Q(F,\ F')$ to itself. The relaxation rate due to absorption R_A is a necessary ingredient of this demonstration. However, the only fact we rely on is the presence in that rate of the 6*j* symbol

$$\begin{Bmatrix}J_l & J_l & K_r\\ 1 & 1 & J_u\end{Bmatrix}.\tag{14}$$

The rate N, in both equations (10) and (11), describes magnetic and HFS depolarization. The importance of this rate is that it accounts for the conversion mechanism of atomic alignment ($K = 2$) into atomic orientation ($K = 1$) discussed by Kemp, Macek, & Nehring (1984). This is related to the fact that, in the algebraic expression of the rate (not given here), K_u and K'_u (see eq. [10]) or K_l and K'_l (see eq. [11]) can have different parity. If the radiation illuminating the atom is not circularly polarized (which is the case of the present discussion), this conversion mechanism is the only process capable of creating orientation in the atomic system (see, e.g., Landolfi & Landi Degl'Innocenti 1985). On the other hand, this mechanism is only effective when quantum interferences between different F levels are important, which corresponds to a regime of magnetic fields such that level crossing between F levels can occur. Therefore, for magnetic fields such that the upper level approaches the regime of the complete Paschen-Back effect ($B > 100$ G), the conversion of atomic alignment into atomic orientation is drastically reduced (see Fig. 2). For this reason, the role of the rate N is not of immediate concern for the following arguments.

We first consider the case in which J_u is at the IP level. When this level is in a regime of very weak coupling between $\boldsymbol{J}$ and $\boldsymbol{I}$ (whether because the HFS separation is much smaller than $A_{J_uJ_l}$ or because a magnetic field is present that is strong enough to establish a regime of complete Paschen-Back effect in that level), the irreducible components of the density matrix for that level ${}^{J_uI}\rho^{K_u}_{Q_u}(F_u,\ F'_u)$ can be written according to equation (8). It is then found that the double summation over F_u and F'_u in equation (11) can be performed algebraically. This corresponds to a contracted product of a 9*j* symbol with three 6*j* symbols, which is evaluated using, e.g., equation (36) of Varshalovich et al. (1988, p. 471). The result is that the overall contribution of the trans-

fer rate T_E to equation (11) is proportional to the product (note that $K_l = K_u$)

$$\begin{Bmatrix} J_l & I & F_l \\ J_l & I & F_l' \\ K_{J_u} & K_I & K_l \end{Bmatrix} \begin{Bmatrix} J_l & J_l & K_{J_u} \\ J_u & J_u & 1 \end{Bmatrix}. \tag{15}$$

Since $J_l < 1$ for the EP level, the former product vanishes unless $K_{J_u} < 2$. In particular, to create alignment in the EP lower level ($K_l = 2$), either both electronic and nuclear orientations ($K_{J_u} = 1,\ K_I = 1,\ 3$) or only nuclear alignment ($K_{J_u} = 0,\ K_I = 2$) must be present when the atom is in the excited state J_u.

To convince ourselves that these conditions cannot be met, let us assume that initially (i.e., before irradiation) atomic polarization is completely absent, in particular, $K_l = 0$. Since the level J_l is EP, it is only sensitive (through the relaxation rate R_A; see the 6*j* symbol in eq. [14]) to the intensity of the incident radiation field, so lower level polarization ($K_l > 0$) cannot be directly created by irradiation. Therefore, when irradiation begins, from equations (10) and (12) we see that the prescribed radiation field ($K_r = 0,\ 2$) can only induce atomic alignment in the upper level (besides populating it), because of the selection rule introduced by the 3*j* symbol in equation (12). Since the atom was initially unpolarized and since by assumption the electronic and nuclear systems are decoupled in the excited state J_u, the atomic alignment of the upper level can only be electronic. In fact, electric-dipole transitions cannot affect the nuclear system, so the nuclear Zeeman sublevels remain naturally populated in all cases of interest, even if strong *J*-*I* coupling is present in the EP level. From this argument, we conclude that $K_{J_u} = 0,\ 2$, and $K_I = 0$ as a result of the excitation process. As anticipated above, we can dismiss the alignment-to-orientation conversion mechanism as a possible source of upper level orientation ($K_{J_u} = 1$) because of the assumed regime of weak *J*-*I* coupling. Also, upper level alignment ($K_{J_u} = 2$) cannot be transferred in the deexcitation process because the product in equation (15) vanishes. Therefore, nuclear polarization can never be created in this regime, and equation (9) applies to the upper level. Under these conditions, the product in equation (15) vanishes identically for $K_l > 0$, so lower level polarization cannot be created. This is in agreement with the results of Paper I and of Figures 1 and 2.

In summary, when the IP upper level is in a regime of very weak *J*-*I* coupling, the creation of atomic alignment in the EP lower level ($K_l = 2$) by transfer of atomic alignment from the IP upper level ($K_{J_u} = 2$) is inhibited. In the case of Na I, this implies that the ground level $S_{1/2}$ cannot be aligned, and consequently, the upper level $P_{1/2}$ of D_1 must also have zero alignment, as illustrated in Paper I and by Figure 1. Lower level orientation ($K_l = 1$) can in principle be created directly by irradiation if $J_l = 1/2$, although it requires that the incident radiation be circularly polarized ($K_r = 1$; see the 6*j* symbol in eq. [14]). In our case, because of the prescribed radiation field, lower level orientation can only be created by the transfer of atomic orientation from the upper level ($K_{J_u} = 1$), which is not inhibited in principle. On the other hand, the alignment-to-orientation conversion mechanism in the upper level becomes very inefficient for very weak *J*-*I* coupling (see Fig. 2), so upper level orientation can also only be created if the incident radiation field is circularly polarized.

We checked our conclusion that equation (9) must apply to the IP upper level in the regime of very weak *J*-*I* coupling against the numerical results of Paper I (see also Fig. 1). In particular, we verified that the ratio of the quantities $\sigma_0^2(3)$ and $\sigma_0^2(1)$ for the upper level $P_{3/2}$ of Na I ($I = 3/2$) in the strong-field limit ($B = 1000$ G; see the rightmost panels of Fig. 1 in Paper I; also Fig. 1) is correctly reproduced by equation (9). This equation also accounts for the curious vanishing of the quantity $\sigma_0^2(2)$ in the same limit, which is due to the fact that $\rho_Q^2(2,\ 2)$ vanishes identically because of the (nontrivial) nullity of the 6*j* symbol in equation (9).

Within the same approximation of the two-level atom ($J_u,\ J_l$), we now assume that the decoupling of $\boldsymbol{J}$ and $\boldsymbol{I}$ is reached first in the lower level, while strong coupling is still present in the upper level. This time we assume that J_l is at the IP level, whereas J_u is at the EP level. Since we assumed that the lower level is in a regime of very weak coupling between $\boldsymbol{J}$ and $\boldsymbol{I}$, the irreducible components of the density matrix for that level ${}^{J_l I}\rho_{Q_l}^{K_l}(F_l,\ F_l')$ can be written according to equation (8). It is then found that the double summation over F_l and F_l' in equation (10) can be performed algebraically. This corresponds to a contracted product of two 9*j* symbols with two 6*j* symbols that is evaluated using, e.g., equation (37) of Varshalovich et al. (1988, p. 471). The result is that the overall contribution of the transfer rate T_A to equation (10) is proportional to the sum

$$\sum_k (-1)^k (2k+1) \begin{Bmatrix} 1 & J_l & J_u \\ 1 & J_l & J_u \\ K_r & K_{J_l} & k \end{Bmatrix} \begin{Bmatrix} I & F_u & J_u \\ I & F_u' & J_u \\ K_I & K_u & k \end{Bmatrix} \begin{Bmatrix} K_r & K_{J_l} & k \\ K_I & K_u & K_l \end{Bmatrix}. \tag{16}$$

Since $J_u < 1$ for the EP level, this sum is limited to $k = 0,\ 1$. Again, we assume that the atomic polarization is absent before irradiation. Because the lower level is IP, lower level alignment can be created when irradiation begins. However, since $\boldsymbol{J}$ and $\boldsymbol{I}$ are decoupled in the lower level, nuclear polarization remains zero ($K_I = 0$), so all the alignment of the lower level must be electronic ($K_{J_l} = 2$). Under these conditions, the sum in equation (16) is restricted to $k = 0$ only, because the first 9*j* symbol vanishes for $k = 1$ unless $K_r + K_{J_l}$ is an odd integer. Therefore, atomic polarization in the upper level ($K_u > 0$) can never be created, because of the nullity of the second 9*j* symbol in equation (16).

This shows, in particular, that the concept of EP is also valid for an upper level. In this case, the EP upper level is sensitive to the anisotropy of radiation ($K_r = 2$) through the transfer rate T_A, but nonetheless, creation of alignment in the upper level through the absorption of that anisotropy is not possible when the IP lower level is in a regime of very weak *J*-*I* coupling, because of the selection rules implied by equation (16). Upper level orientation ($K_u = 1$) is not excluded in principle if $J_u = 1/2$, although it can only be created by transfer of atomic orientation from the lower level ($K_{J_l} = 1$; see eq. [16]). However, when the lower level is in the regime of weak *J*-*I* coupling, its orientation can only be due to the presence of circular polarization in the incident radiation field.

4. CONCLUSIONS

In this paper we analytically demonstrated that the presence of *J-I* coupling in the IP level $P_{3/2}$ of Na I is a necessary condition for the transfer of atomic alignment from that level to the EP ground level $S_{1/2}$. We based our demonstration on the quantum theory of line formation as developed by Landi Degl'Innocenti (1983, 1984, 1985), assuming unpolarized incident radiation without spectral structure over the frequency intervals encompassing the HFS components of the atomic transitions of interest. Under these conditions, we relied on the argument that nuclear polarization cannot be created in an atom having only one IP level if $\boldsymbol{J}$ and $\boldsymbol{I}$ are completely decoupled in that level, because the assumed incident radiation cannot directly induce nuclear transitions in the atom.

It follows, from the results of § 3, that atomic polarization cannot be created in the EP levels when $\boldsymbol{J}$ and $\boldsymbol{I}$ are completely decoupled in the IP level.

We can further strengthen this argument by showing that the possibility of nuclear polarization actually resides in the presence of *J-I* coupling in the IP level, whereas the presence of *J-I* coupling in the EP level is not relevant. To this purpose, we repeated the calculation of Figure 1 after artificially zeroing the HFS separation in the level $S_{1/2}$ of Na I. The results of this calculation are shown in Figure 3. Since $\boldsymbol{J}$ and $\boldsymbol{I}$ are completely decoupled in the "modified" level $S_{1/2}$, the factorization in equation (6) always applies to this level. On the other hand, any atomic alignment in this modified EP level requires the presence of nuclear polarization (see eq. [8]), since the electronic angular momentum of the level is $J = 1/2$. Such nuclear polarization in the EP level $S_{1/2}$ can only come from the atomic polarization of the IP level $P_{3/2}$ (which is transferred to the EP level via radiative deexcitation), since it is not possible for the prescribed radiation field to directly create atomic polarization in the EP level. From the results of Figure 3, it is evident that the nuclear polarization in the modified level $S_{1/2}$ vanishes when the regime of the complete Paschen-Back effect is reached in the level $P_{3/2}$ and equation (6) also applies to that level. Comparing the results of Figures 1 and 3, we see that the suppression of *J-I* coupling in the level $S_{1/2}$ does not substantially alter the SE of the model atom. On the basis of these arguments, it seems safe to conclude that, even in the real case, nuclear polarization cannot be created in the atom when the regime of the complete Paschen-Back effect is reached in the (only) IP level.

Finally, we must emphasize that the presence of atomic alignment in the upper level of the D_1 line induces a characteristic *antisymmetric* signature in the core of the Stokes Q profile resulting from the scattering of the anisotropic radiation illuminating the atom (see Fig. 2 of Paper I). This applies particularly to the optically thin "prominence case" considered in Paper I, in which the scattering polarization is solely due to the emission events following atomic excitation by the anisotropic radiation. Currently, we are investigating to what extent such an antisymmetric signature can be modified through dichroism and radiative transfer effects, because of the presence of atomic alignment in the ground level of Na I (see, e.g., Trujillo Bueno & Landi Degl'Innocenti 1997; for the observable effects of dichroism and ground-level polarization on the He I 10830 Å multiplet, see Trujillo Bueno et al. 2002b).

In this respect, it is interesting to note that spectropolarimetric observations of the Na I D lines obtained with THÉMIS[6] in quiet regions close to the solar limb show an antisymmetric signature in the fractional linear polarization Q/I of the D_1 line (see Fig. 1 of Trujillo Bueno & Manso Sainz 2001, which was adapted from Trujillo Bueno et al. 2001; see also Bommier & Molodij 2002). There seems to be an indication of a similar antisymmetric signature in the Q/I atlas of Gandorfer (2000), which was obtained with the polarimeter ZIMPOL-II attached to the Gregory Coudé Telescope (GCT) of the Istituto di Ricerche Solari at Locarno (Switzerland). On the contrary, analogous observations that Stenflo, Gandorfer, & Keller (2000) had obtained previously with the polarimeter ZIMPOL-I attached to the McMath-Pierce facility of the National Solar Observatory show almost symmetric profiles with a central positive peak (see their Fig. 3).[7]

[6] THÉMIS is a polarization-free solar telescope operated by CNRS-CNR in the Spanish Observatorio del Teide of the Instituto de Astrofísica de Canarias.

[7] These Q/I observations of quiet solar regions were obtained in 1998 March (i.e., 2 years earlier than the above mentioned THÉMIS observations), when the Sun had not yet reached the maximum of its magnetic activity cycle.

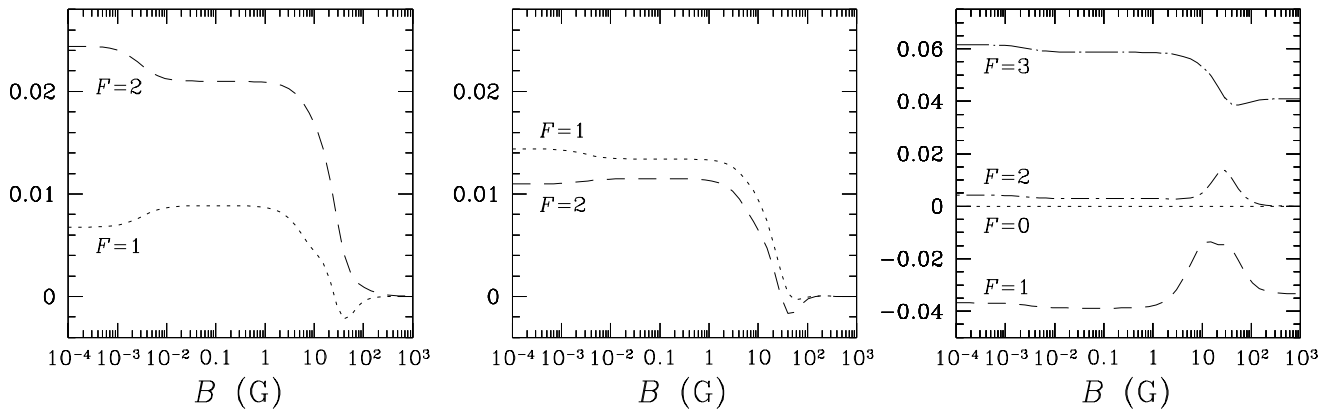

FIG. 3.—Same as Fig. 1, but assuming that no HFS is present in the ground level $S_{1/2}$. These results show that transfer of atomic alignment from the IP level $P_{3/2}$ to the EP level $S_{1/2}$ still occurs, even if $\boldsymbol{J}$ and $\boldsymbol{I}$ are completely decoupled in the EP level, so far as *J-I* coupling is present in the IP level.

As shown in Paper I, for single-scattering events, one should expect a *symmetric* shape of the Stokes Q signature in the core of the D_1 line for magnetic fields $B \gtrsim 50$ G (see Fig. 2 of Paper I; note that such a symmetric signature would change its sign if we considered, e.g., a horizontal canopy-like field instead of the vertical field assumed for the calculation of that figure). Nevertheless, we think that the above-mentioned linear-polarization observations of the D_1 line in very quiet regions of the solar disk with THÉMIS and ZIMPOL both have the same physical origin, i.e., atomic alignment in the levels of the Na I D_1 line. Now that we understand how the ground level of Na I becomes polarized and how its polarization is modified by the presence of weak magnetic fields, it will be worthwhile to investigate the sodium polarization problem by means of full radiative transfer simulations including collisions, taking also into account the quantum interferences among the two upper levels of the "enigmatic" Na I D lines.

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REFERENCES

Bommier, V., & Molodij, G. 2002, A&A, 381, 241
Gandorfer, J. 2000, The Second Solar Spectrum: A High Spectral Resolution Polarimetric Survey of Scattering Polarization at the Solar Limb in Graphical Representation (Zurich: vdf Hochschulverlag AG an der ETH)
Kemp, J. C., Macek, J. H., & Nehring, F. W. 1984, ApJ, 278, 863
Landi Degl'Innocenti, E. 1983, Sol. Phys., 85, 3
———. 1984, Sol. Phys., 91, 1
———. 1985, Sol. Phys., 102, 1
———. 1998, Nature, 392, 256
Landolfi, M., & Landi Degl'Innocenti, E. 1985, Sol. Phys., 98, 53
Lehmann, J. C. 1969, Phys. Rev., 178, 153
Stenflo, J. O. 2001, in ASP Conf. Ser. 236, Advanced Solar Polarimetry: Theory, Observation, and Instrumentation, ed. M. Sigwarth (San Francisco: ASP), 97
Stenflo, J. O., Gandorfer, A., & Keller, C. U. 2000, A&A, 355, 781
Trujillo Bueno, J. 2001, in ASP Conf. Ser. 236, Advanced Solar Polarimetry: Theory, Observation, and Instrumentation, ed. M. Sigwarth (San Francisco: ASP), 161
Trujillo Bueno, J., Casini, R., Landolfi, M., & Landi Degl'Innocenti, E. 2002a, ApJ, 566, L53 (Paper I)
Trujillo Bueno, J., Collados, M., Paletou, F., & Molodij, G. 2001, in ASP Conf. Ser. 236, Advanced Solar Polarimetry: Theory, Observation, and Instrumentation, ed. M. Sigwarth (San Francisco: ASP), 141
Trujillo Bueno, J., & Landi Degl'Innocenti, E. 1997, ApJ, 482, L183
Trujillo Bueno, J., Landi Degl'Innocenti, E., Collados, M., Merenda, L., & Manso Sainz, R. 2002b, Nature, 415, 403
Trujillo Bueno, J., & Manso Sainz, R. 2001, in ASP Conf. Ser. 248, Magnetic Fields across the Hertzsprung-Russell Diagram, ed. G. Mathys, S. K. Solanki, & D. T. Wickramasinghe (San Francisco: ASP), 83
———. 2002, Il Nuovo Cimento C, in press
Varshalovich, D. A., Moskalev, A. N., & Khersonskii, V. K. 1988, The Quantum Theory of Angular Momentum (Singapore: World Scientific)